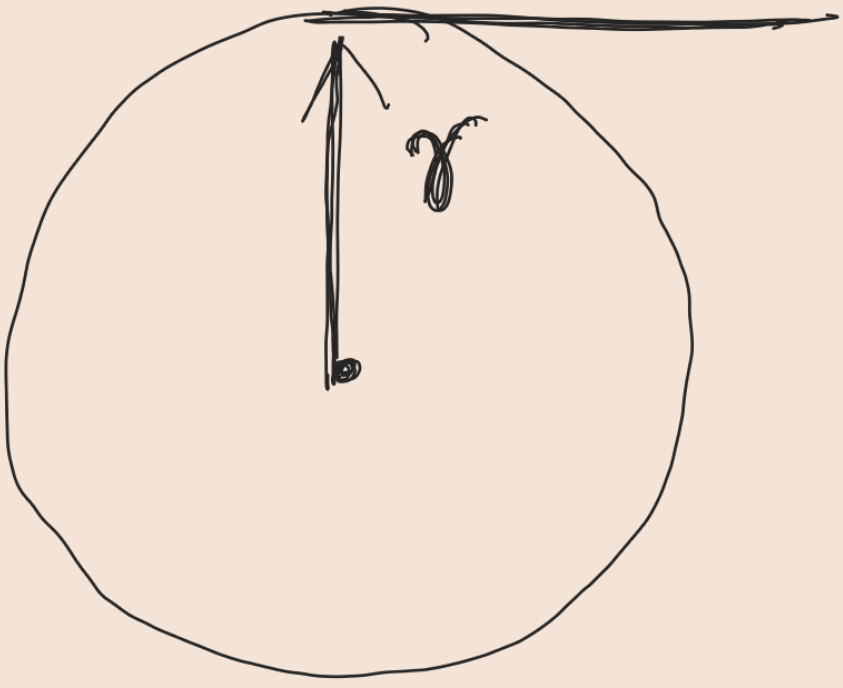
t' = tape thickness



$$dx = r\,d\theta$$

$$(dx)\,t' = (2\pi r)(dr).$$

$$\frac{l}{L} = \frac{r^2 - r_0^2}{R^2 - r_0^2}$$

Also, $v = rw$ Hence,

$$\left(\frac{dx}{dt}\right) t' = rw.$$

Substituting, we have $\left(\frac{dx}{dt}\right) t' = 2\pi r \frac{dr}{dt}$

$$\frac{\gamma\omega}{t'} = (2\pi\gamma)\frac{dr}{dt}.$$

$$\boxed{\frac{t}{T} = \frac{r - r_0}{R - r_0}} \quad (\text{As } \omega = \text{Constant})$$

$$r = \left(\frac{t}{T}\right)(R - r_0) + r_0 \quad \therefore$$

$$l = L\left[\left(\left(\frac{t}{T}\right)(R - r_0) + r_0\right)^2 - r_0^2\right]$$

$$R^2 - r_0^2$$

In this approach, we have equated elemental volume of outflow of tape i's' elemental volume decrease. The question is a bridge between discrete & continuous layer addition because tape thickness t' is tending to zero.